Chapter 9

Presto-digit-ation: The Art of Mathematical Magic

Playing with numbers has brought me great joy in life. I find that arithmetic can be just as entertaining as magic. But understanding the magical secrets of arithmetic requires algebra. Of course, there are other reasons to learn algebra (SATs, modeling real-world problems, computer programming, and understanding higher mathematics, to name just a few), but what first got me interested in algebra was the desire to understand some mathematical magic tricks, which I now present to you!

PSYCHIC MATH

Say to a volunteer in the audience, "Think of a number, any number." And you should also say, "But to make it easy on yourself, think of a one-digit or two-digit number." After you've reminded your volunteer that there's no way you could know her number, ask her to:

1. double the number,
2. add 12,

3. divide the total by 2,
4. subtract the original number.

Then say, "Are you now thinking of the number six?" Try this one on yourself first and you will see that the sequence always produces the number 6 no matter what number is originally selected.

Why This Trick Works

This trick is based on simple algebra. In fact, I sometimes use this as a way to introduce algebra to students. The secret number that your volunteer chose can be represented by the letter x. Here are the functions you performed in the order you performed them:

1. $2x$ (double the number)
2. $2x + 12$ (then add 12)
3. $(2x + 12) \div 2 = x + 6$ (then divide by 2)
4. $x + 6 - x = 6$ (then subtract original number)

So no matter what number your volunteer chooses, the final answer will always be 6. If you repeat this trick, have the volunteer add a different number at step 2 (say 18). The final answer will be half that number (namely 9).

THE MAGIC 1089!

Here is a trick that has been around for centuries. Have your audience member take out paper and pencil and:

1. secretly write down a three-digit number where the digits are decreasing (like 851 or 973),

2. reverse that number and subtract it from the first number,
3. take that answer and add it to the reverse of itself.

At the end of this sequence, the answer 1089 will magically appear, no matter what number your volunteer originally chose. For example:

$$\begin{array}{r} 851 \\ -\ 158 \\ \hline 693 \\ +\ 396 \\ \hline 1089 \end{array}$$

Why This Trick Works

No matter what three-digit number you or anyone else chooses in this game, the final result will always be 1089. Why? Let *abc* denote the unknown three-digit number. Algebraically, this is equal to:

$$100a + 10b + c$$

When you reverse the number and subtract it from the original number, you get the number *cba,* algebraically equal to:

$$100c + 10b + a$$

Upon subtracting *cba* from *abc,* you get:

$$\begin{aligned} &100a + 10b + c - (100c + 10b + a) \\ &= 100(a - c) + (c - a) \\ &= 99(a - c) \end{aligned}$$

Hence, after subtracting in step 2, we must have one of the following multiples of 99: 198, 297, 396, 495, 594, 693, 792, or 891, each one of which will produce 1089 after adding it to the reverse of itself, as we did in step 3.